SCET and small-x physics: comments and ideas

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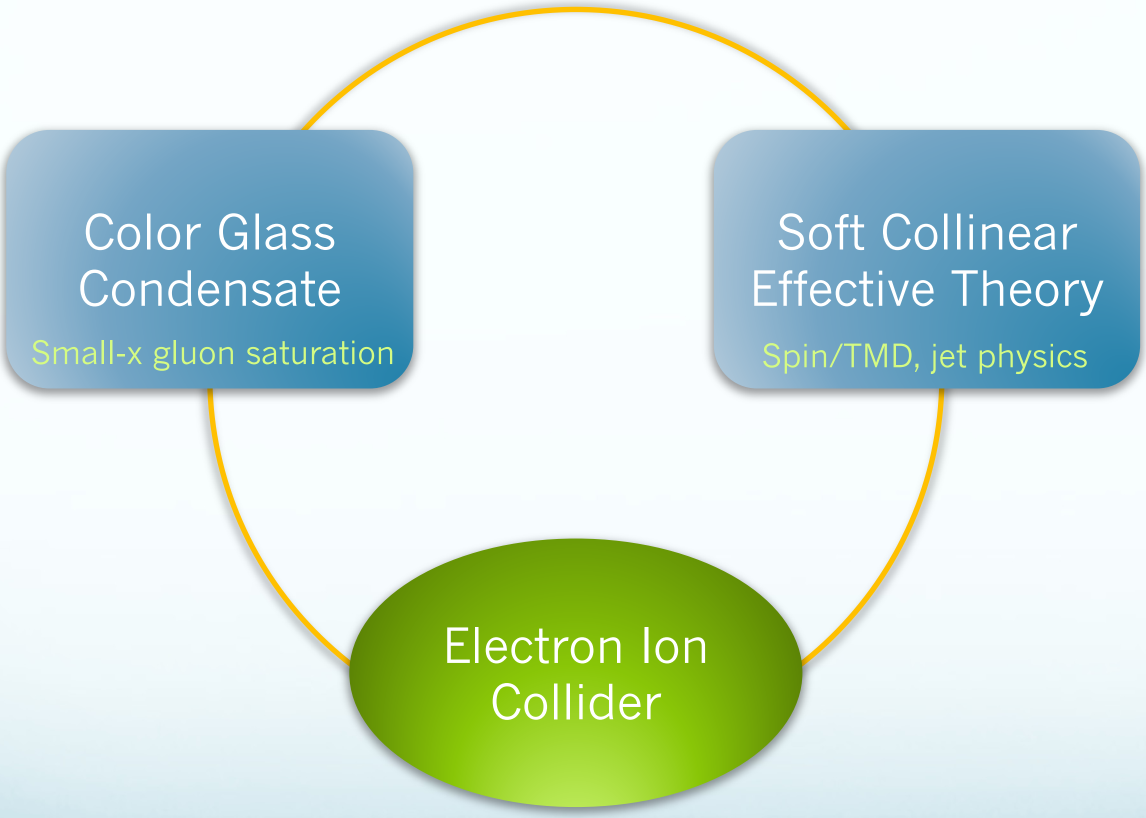
Small-x/CGC and SCET

- Utilizing and combing two frameworks to make efficient computations for small-x observables, while with EIC in mind



Soft Collinear Effective Theory (SCET)

- SCET: an effective theory for highly energetic partons interacting with collinear and/or soft gluons

- Highly energetic quark/gluon jets are described "collinear" fields

$$k \sim \left[k^+, k^-, k_\perp\right]$$

Hard: $k \sim Q\,[1,1,1]$

Soft: $k \sim Q\,[\lambda,\lambda,\lambda]$

Collinear: $k \sim Q\left[1,\lambda^2,\lambda\right]$

Bauer et al. 01, Pirol et al. 04

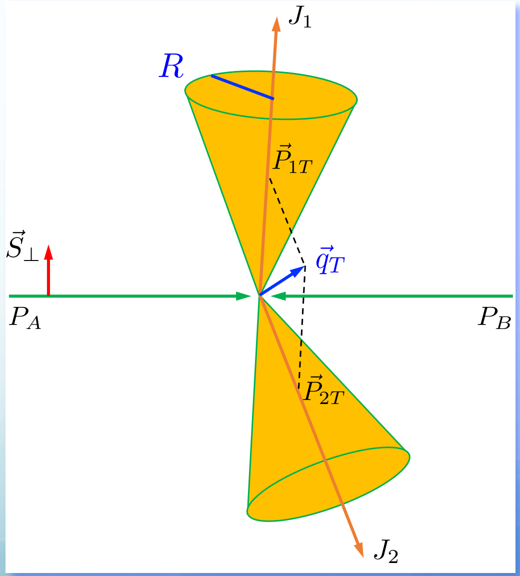
Collinear quarks, antiquarks	$\xi_n, \bar{\xi}_n$
Collinear gluons, soft gluons	A_n, A_s

- Back-to-back dijet production

$$\vec{q}_T = \vec{P}_{1T} + \vec{P}_{2T}$$

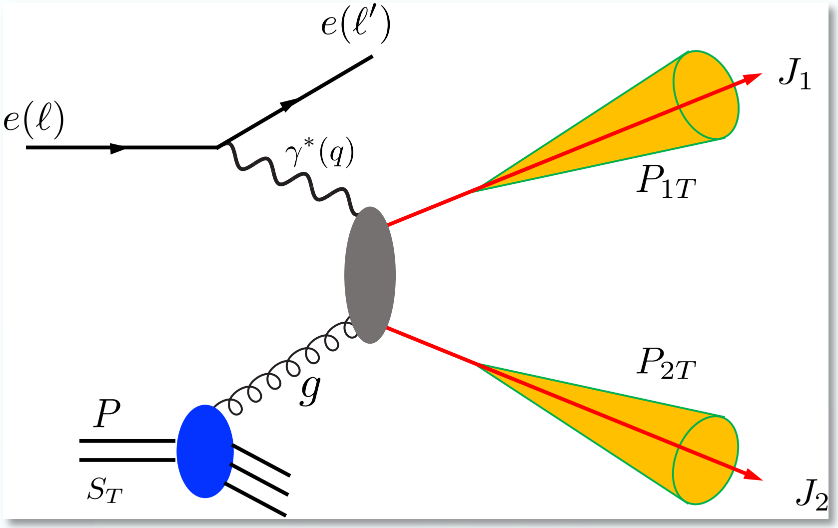
$$\vec{P}_T = \frac{\vec{P}_{1T} - \vec{P}_{2T}}{2} \qquad \lambda = \frac{q_T}{P_T}$$

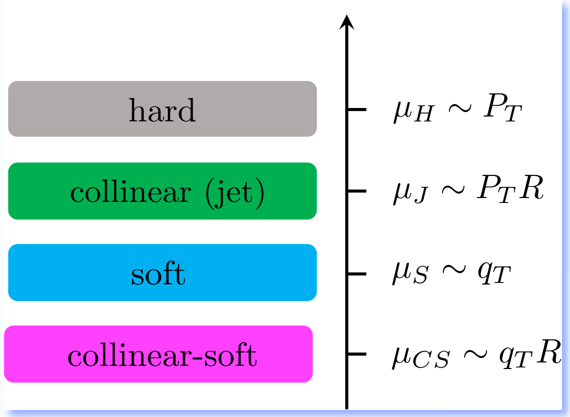
$$q_T \ll P_T$$



SCET: factorization

- Factorization is simple: back-to-back dijet production at EIC



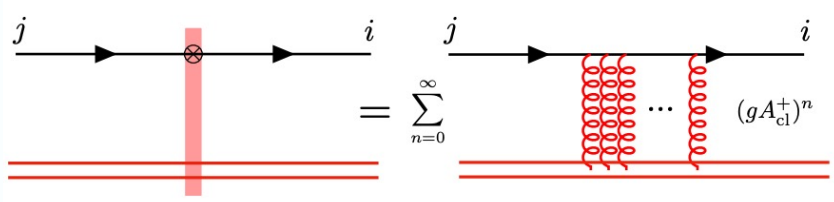


$$\frac{d\sigma}{dq_T dP_T} \propto \boldsymbol{H}_{\gamma * g \to q\bar{q}}(P_T) f_{g/A}(x_a, k_{aT}) \otimes \boldsymbol{S}^{\text{global}}(q_T) \otimes S_c^{\text{cs}}(q_T R) S_d^{\text{cs}}(q_T R) J_c(P_T R) J_d(P_T R)$$

✓ You can quickly convince yourself that such formalism: (1) can reproduce the result of Sudakov resummation, (2) it is much more accessible than the Sudakov resummation

CGC: formalism

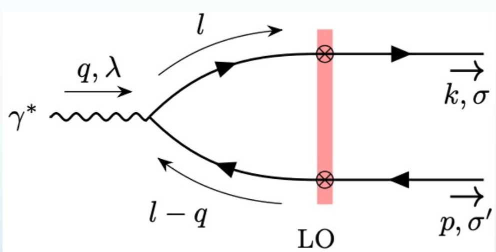
- CGC: shockwave

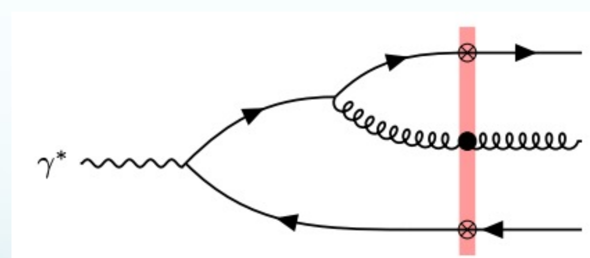


$$\mathcal{T}^q_{ij}(l',l) = (2\pi)\delta(l^- - l'^-)\gamma^-\mathrm{sgn}(l^-)\int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{l}'_\perp-\boldsymbol{l}_\perp)\cdot\boldsymbol{x}_\perp} V^{\mathrm{sgn}(l^-)}_{ij}(\boldsymbol{x}_\perp)$$

$$V_{ij}(\boldsymbol{x}_\perp) = \mathcal{P}\ \exp\left(ig\int_{-\infty}^{\infty} \mathrm{d}z^- A^{+,a}_{\mathrm{cl}}(z^-,\boldsymbol{x}_\perp)t^a_{ij}\right)$$

- Dijet production in e+A collisions





Lots of progress on CGC

- Most of the time, we concentrate on deriving the small-x evolution equation: a lot of progress (*not a complete list*)
  - BFKL (NNLO?): linear in ln(1/x)
  - BK (NLO): nonlinear + running coupling
  - JIMWLK (NLO)
  - The implementation/solution of these equations at NLO or beyond is always quite tricky: instability issue

- However, the computation on the physical observables beyond LO is still limited (*not a complete list*)
  - Forward hadron in p+A at NLO
    - Chirilli, Xiao, Feng, 2012, many other follow-up, most recent: Xiao, et.al. 2112.06975
    - Kang, Liu, 1910.10166, 2004.11990
  - Photon in p+A at NLO
    - Benic, Fukushima, Garcia-Montero, Venugopalan, 17
  - DIS at NLO
    - Beuf, Hanninen, Lappi, Mantysaari, 2007.01645
    - Balitsky, Chirilli 12, Beuf 11
  - Photon, photon+dijet at DIS
    - Roy, Venugopalan 18, 19
  - dijet at DIS
    - Caucal, Salazar, Venugopalan, 21

Status

- In general, these results are extremely complicated

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Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate

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ABSTRACT: We compute the next-to-leading order impact factor for inclusive dijet production in deeply inelastic electron-nucleus scattering at small x_{Bj}. Our computation, performed in the framework of the Color Glass Condensate effective field theory, includes all real and virtual contributions in the gluon shock wave background of all-twist lightlike Wilson line correlators. We demonstrate explicitly that the rapidity evolution of these correlators, to leading logarithmic accuracy, is described by the JIMWLK Hamiltonian. When combined with the next-to-leading order JIMWLK Hamiltonian, our results for the impact factor improve the accuracy of the inclusive dijet cross-section to $\mathcal{O}(\alpha_s^2 \ln(x_f/x_{\mathrm{Bj}}))$, where x_f is a rapidity factorization scale. These results are an essential ingredient in assessing the discovery potential of inclusive dijets to uncover the physics of gluon saturation at the Electron-Ion Collider.

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NLO impact factor for inclusive photon + dijet production in $e+A$ DIS at small x

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We compute the next-to-leading order (NLO) impact factor for inclusive photon + dijet production in electron-nucleus (e + A) deeply inelastic scattering (DIS) at small x. An important ingredient in our computation is the simple structure of "shock wave" fermion and gluon propagators. This allows one to employ standard momentum space Feynman diagram techniques for higher order computations in the Regge limit of fixed $Q^2 \gg \Lambda_{\mathrm{QCD}}^2$ and $x \to 0$. Our computations in the color glass condensate (CGC) effective field theory include the resummation of all-twist power corrections Q_s^2/Q^2, where Q_s is the saturation scale in the nucleus. We discuss the structure of ultraviolet, collinear and soft divergences in the CGC, and extract the leading logs in x; the structure of the corresponding rapidity divergences gives a nontrivial first principles derivation of the JIMWLK renormalization group evolution equation for multiparton lightlike Wilson line correlators. Explicit expressions are given for the x-independent $O(\alpha_s)$ contributions that constitute the NLO impact factor. These results, combined with extant results on NLO JIMWLK evolution, provide the ingredients to compute the inclusive photon + dijet cross section at small x to $O(\alpha_s^3 \ln(x))$. First results for the NLO impact factor in inclusive dijet production are recovered in the soft photon limit. A byproduct of our computation is the LO photon + 3 jet (quark-antiquark-gluon) cross section.

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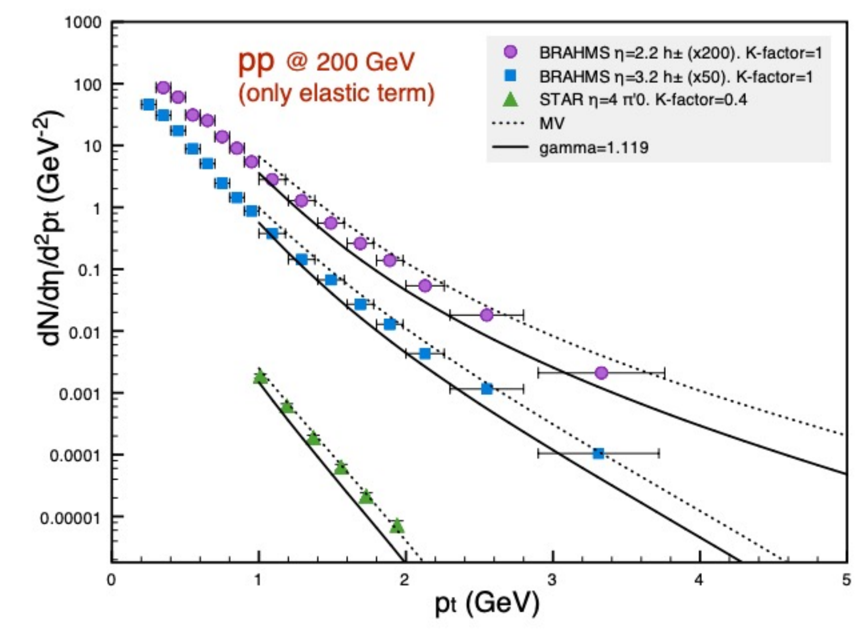
✓ Maybe the results would be simpler in some kinematic limit, where SCET or other EFTs can be used to derive the formalism
✓ Also other lessons can be learned from each other in different communities

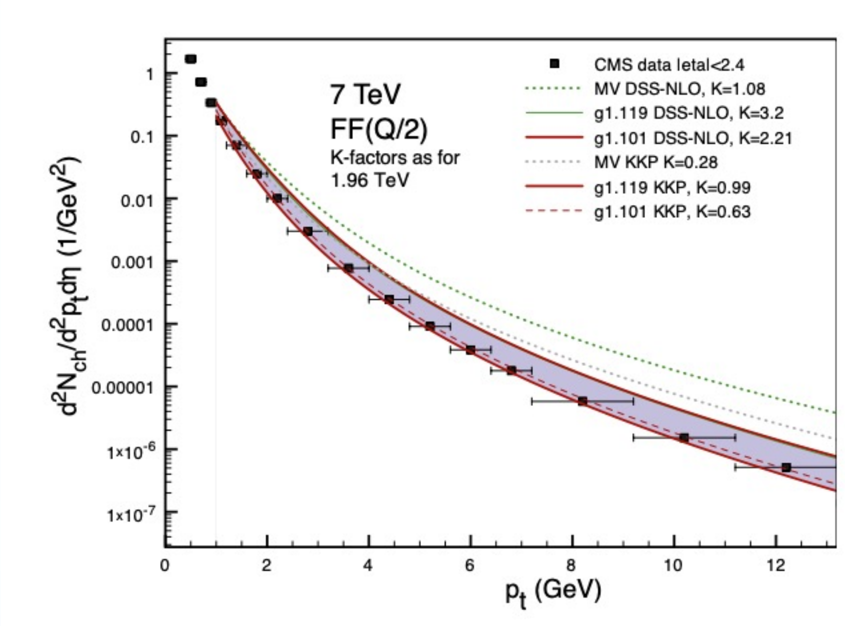
Outstanding questions in CGC

- At NLO: how to naturally set the rapidity scale? [*is this even a thing?*]
  - For both NLO hard functions + the scale for UGD BK-JMWLK evolution, need to be the same
  - How to efficiently perform resummation in small-x formalism, e.g., threshold resummation?
- At NLO: how does the CGC formalism reduce to TMD factorization in the correlation limit?
  - E.g., for back-to-back dijet production at e+A collisions at LO, consistency was shown between CGC and TMD (Xiao, Yuan, Mueller et.al.), what about NLO?
  - Can one re-express the CGC results in a factorized form, e.g., UGD * soft function * collinear-soft function * jet function type?
  - If this is possible, we can easily generalize to other process, e.g., dijet, photon+jet, heavy flavor dijet in p+A collisions (or at UPC), instead of doing complicated computation each time?
- How could one perform joint TMD and BK evolution equation?
  - There are work along this direction. However, is it possible to find connection between those well-fitted gluon TMD and the MV model/GBW model: maybe as an input at relatively large x?
  - If this is possible, one might be able to perform global analysis for small-x observables cross all different processes, to extract universal set of UGD?
- Question: use Glauber modes from EFT to derive small-x RG equation or computations
  - It is also important questions, but might not be immediately relevant for small-x phenomenology yet

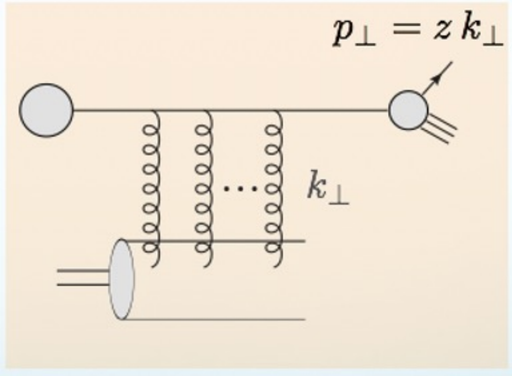
Don't know if NLO would work fine

- Single hadron production in p+A collisions: LO works fine



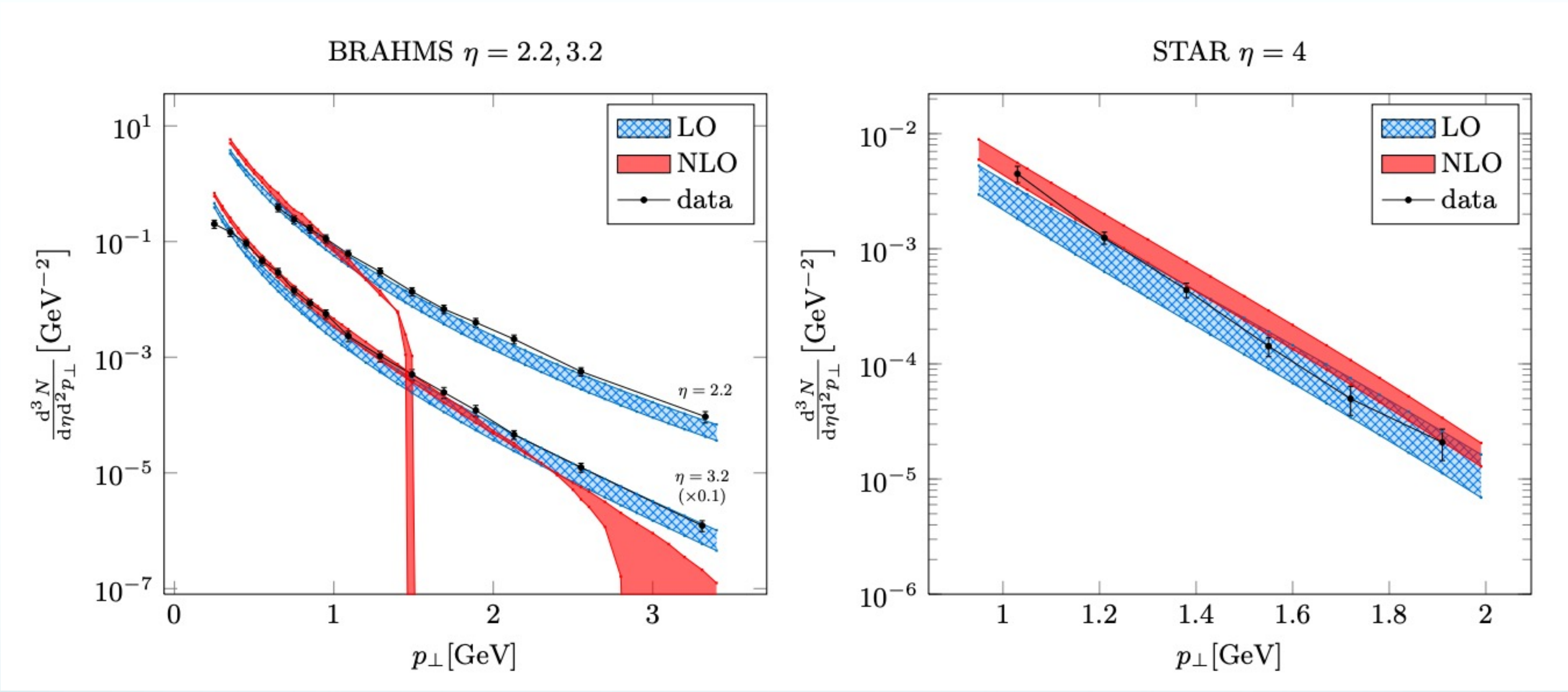


Albaceta, Dumitru, Fujii, Nara, 1209.2001



But naïve NLO does not

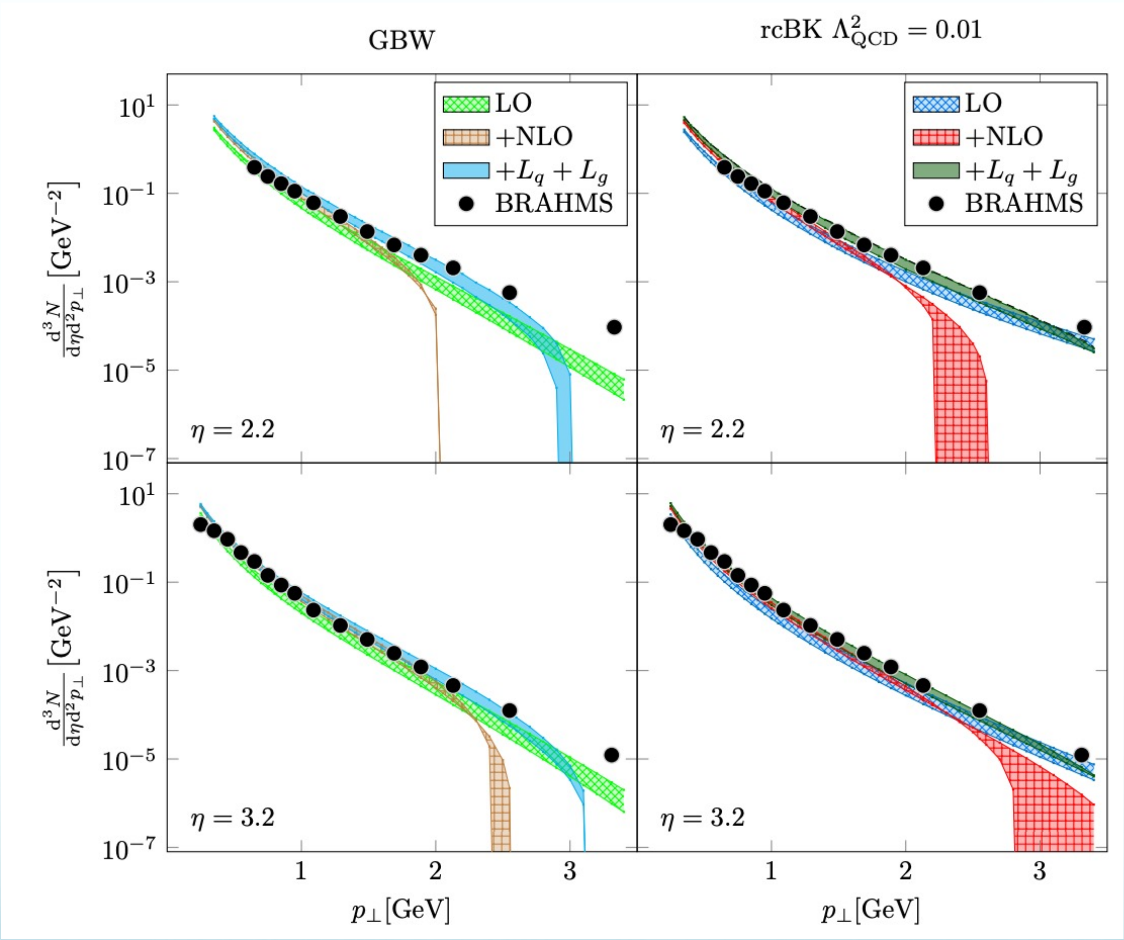
- NLO computations
  - quickly becomes negative



Stasto, Xiao, Zaslavsky, 1307.4057

Adding so-called kinematic constraints helps

- So-called kinematic constraints give some positive contributions, but not enough

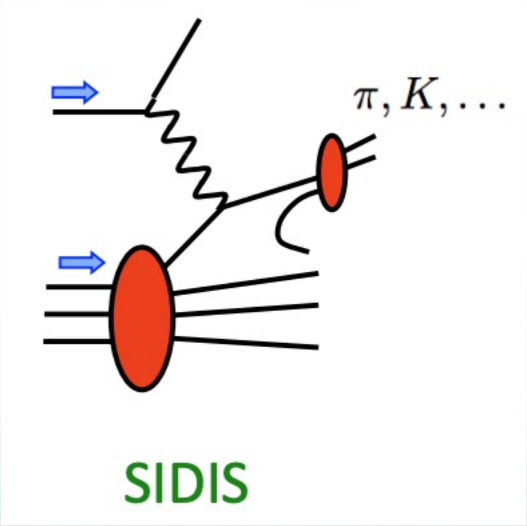


Watanabe, Xiao, Yuan, Zaslavsky, 1505.05183

Issues?

- There seems to be some issues for NLO computations
  - To solve the negative cross sections, as well as some theory issues
- It is better to review a NLO calculations in the dilute collinear factorization
  - Using SIDIS as an example:

$$e + p \to e + h + X$$

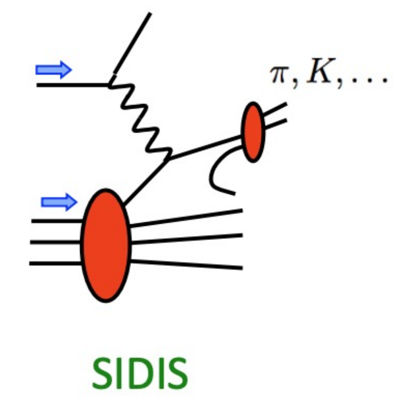


LO + NLO

- SIDIS at LO and NLO:

$$\frac{d\sigma^{(0)}}{dx_B dQ^2 dz_h} = \sigma_0 \sum e_q^2 f_{q/p}(x_B) D_{h/q}(z_h)$$

$$\begin{aligned}\frac{d\sigma^{(1)}}{dx_B dQ^2 dz_h} =& \sigma_0 \sum e_q^2 \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} f_{q/p}(x) D_{h/q}(z) \\ &\times \frac{\alpha_s}{2\pi} \left\{ \left[-\frac{1}{\hat{\epsilon}} + \ln\left(\frac{Q^2}{\mu^2}\right) \right] P_{qq}(\hat{z}) \delta(1-\hat{x}) \right. \\ &\left. + \left[-\frac{1}{\hat{\epsilon}} + \ln\left(\frac{Q^2}{\mu^2}\right) \right] P_{qq}(\hat{x}) \delta(1-\hat{z}) \right\}\end{aligned}$$



- Key:
  - You will encounter divergences, you'd better regularize them properly
    - Standard dimensional regularization is enough: maintain Lorentz invariance etc

Key continues

- Key
  - These divergences will be absorbed into the redefinition of the collinear PDFs and collinear FFs, and thus lead to DGLAP evolution of these distributions

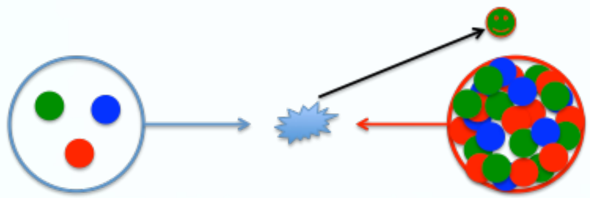
$$f_{q/p}(x_B, \mu) = f_{q/p}(x_B) + \frac{\alpha_s}{2\pi}\left[-\frac{1}{\hat{\epsilon}}\right]\int_{x_B}^{1}\frac{dx}{x}f_{q/p}(x)P_{qq}(\hat{x} = x_B/x)$$

$$D_{h/q}(z_h, \mu) = D_{h/q}(z_h) + \frac{\alpha_s}{2\pi}\left[-\frac{1}{\hat{\epsilon}}\right]\int_{z_h}^{1}\frac{dz}{z}D_{h/q}(z)P_{qq}(\hat{z} = z_h/z)$$

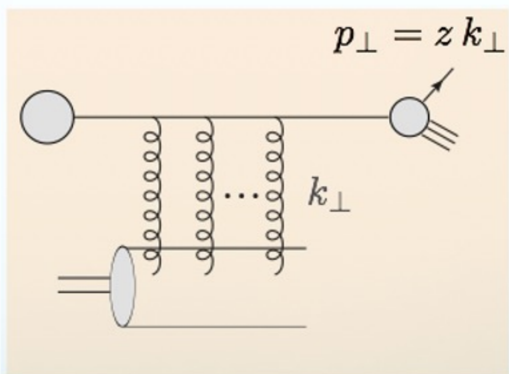
  - You will know what hard scale you are going to use
    - **ln(Q^2/μ^2) suggests that one can set Q=μ as the natural hard scale**
    - Thus one removes the potential large logarithms in the hard function, while evolve your PDFs/FFs to the hard scale Q

Single hadron production in dense regime

- Single hadron in pA at LO in the forward region
  - A hybrid formalism: collinear PDFs for proton, CGC formalism for the nucleus



$$x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim \mathcal{O}(1)$$
$$x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$$



Coherent multiple scattering as in small-x/CGC formalism

$$\frac{d\sigma}{dy d^2p_\perp} = \int_\tau^1 \frac{dz}{z^2} D_{h/q}(z) x_p f_{q/p}(x_p) \mathcal{F}(x_g, k_\perp)$$

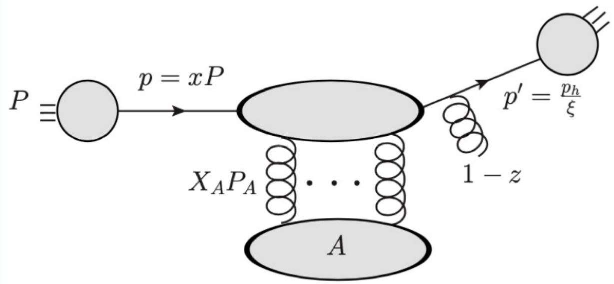
$$\mathcal{F}(x_g, k_\perp) = \int \frac{d^2b_\perp d^2b'_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(b_\perp - b'_\perp)} S^{(2)}(b_\perp, b'_\perp),$$

$$S^{(2)}(b_\perp, b'_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(b_\perp)U^\dagger(b'_\perp)\right]\right\rangle$$

$$U(b_\perp) = \mathcal{P}\exp\left\{ig_s\int_{-\infty}^{+\infty} dx^+ t^c A_c^-(x^+, b_\perp)\right\}$$

NLO formalism: initial

- NLO: gluon radiation



- Possible divergences: collinear divergences for collinear PDFs+FFs, just like SIDIS we have discussed (DGLAP evolution)
  - Standard dimensional regularization
  - You know the natural hard scale pT of the hadron
- Additional divergence: rapidity divergence for unintegrated gluon distribution (BK evolution)

$$d\sigma^{(1)} \propto \int^1 dz \left(-\frac{1}{\epsilon} P^{(1)}_{i\to j}(z) - \frac{1}{\epsilon}\frac{1}{z^2} P^{(1)}_{j\to k}(z) - P^{(1)}_{i\to j}(z) \ln\frac{\mu^2}{p_\perp^2} + \dots \right) + \frac{1}{1-z} \left(\kappa^{(1)} \otimes \mathcal{F}_A - \alpha_s \left[\mathcal{F}_A\right]^2 \right) +$$

BK kernel

Check closely

- The rapidity divergence should be absorbed into the unintegrated gluon distribution of the nucleus
  - Thus leads to the rapidity Y-evolution of BK equation: $Y=\ln(1/x_g)$

$$\frac{\partial}{\partial Y}S_Y^{(2)}(x_\perp,y_\perp)=-\frac{\alpha_s N_c}{2\pi^2}\int\frac{d^2b_\perp\,(x_\perp-y_\perp)^2}{(x_\perp-b_\perp)^2(y_\perp-b_\perp)^2}\left[S_Y^{(2)}(x_\perp,y_\perp)-S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\right]$$

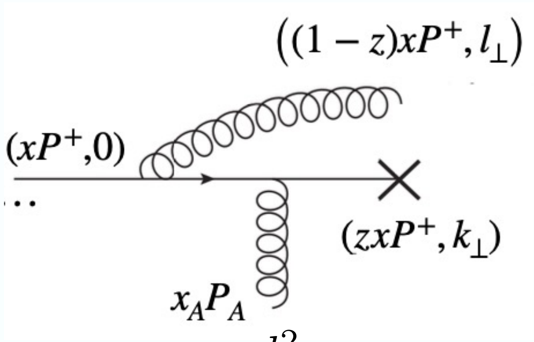
  - The question is: where does the Y arise at all?
  - The divergence is not regularized, so how exactly do you do the absorption?

$$\int_0^1\frac{dz}{1-z}[\mathcal{K}_{\mathrm{BK}}]$$

It is important to note that if we conduct the leading order classical calculation, we will not get any energy dependence, namely the Y dependence, in the scattering amplitudes. It is the BK evolution equation as shown above which gives the energy dependence to those scattering amplitudes. To derive the BK equation from Eqs. (6, 21), one needs to reset the upper limit of the $d\xi$ integral in Eq. (21) to $1-e^{-Y}$, with Y being the total rapidity gap between the projectile proton and the target nucleus. Here $Y\to\infty$ as the center of mass energy $s\to\infty$. By doing so, we introduce the rapidity Y dependence, namely the energy dependence, of the two-point function $S_Y^{(2)}(x_\perp,y_\perp)$ from which the BK equation can be understood and therefore derived. Another way to derive this equation is to slightly move away from

How to find that Y then?

- Kinematic constraint



$$x_g P^- = \frac{l_\perp^2}{2(1-z)xP^+} + \frac{k_\perp^2}{2zxP^+} \le P^-$$

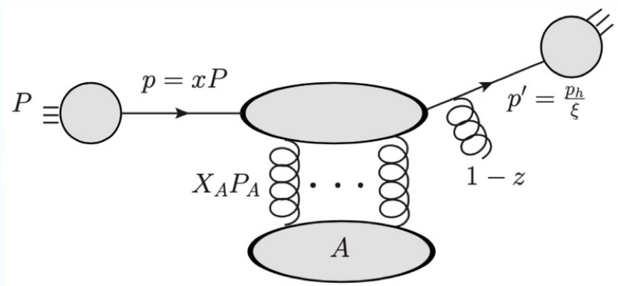
$$\to z \le 1 - \frac{l_\perp^2}{xs}$$

$$\int_0^{1-\frac{l_\perp^2}{xs}} \frac{dz}{1-z} = \ln(1/x_g) + \ln\left(\frac{k_\perp^2}{l_\perp^2}\right)$$

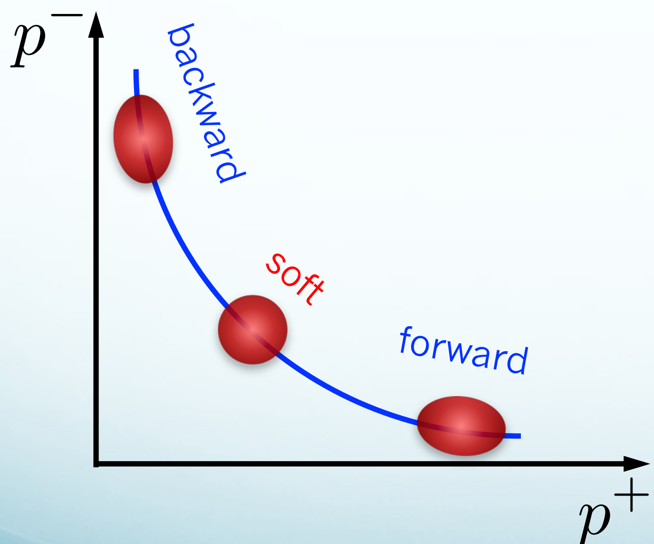
- This seems to be nice, but
  - One mix the power counting: kinematic constraints seem to be power-suppressed, while the previous calculation is leading power (eikonal approx.)
  - It is quite difficult to implement universally or at even higher order

Power counting for the observable

- Apparently both collinear and soft modes contribute to the pT spectrum, in the semi-hard region



$$\lambda \sim \mathcal{O}\left(\frac{p_\perp}{\sqrt{s}}\right) \ll 1$$



Modes contribute to $p_\perp$

$(+,\perp,-)$

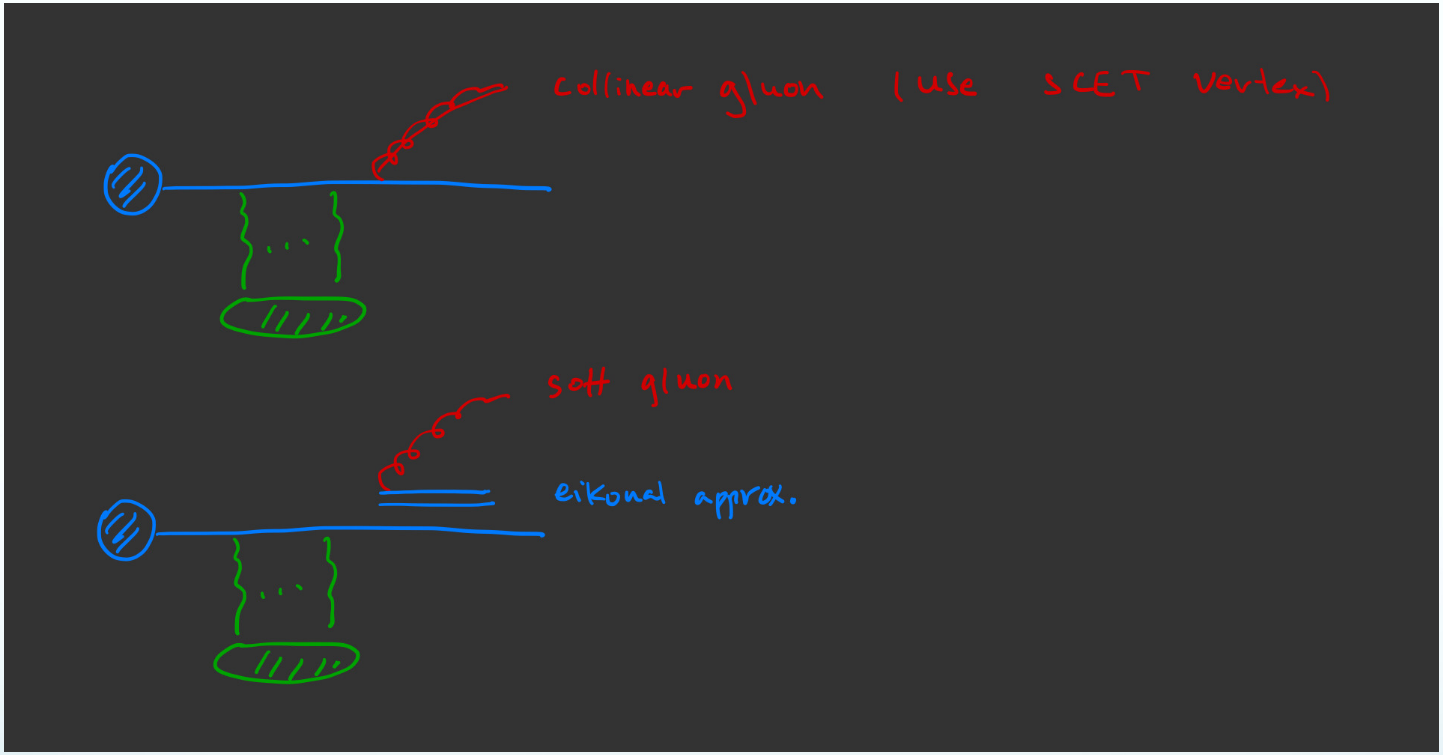
$$P_c = \frac{\sqrt{s}}{2}(1,\lambda,\lambda^2) \qquad z \sim 1-z \sim \mathcal{O}(1) \quad \text{forward rapidity}$$

$$P_{\bar{c}} = \frac{\sqrt{s}}{2}(\lambda^2,\lambda,1)$$

$$k_s = \frac{\sqrt{s}}{2}(\lambda,\lambda,\lambda) \qquad 1-z \sim \mathcal{O}(\lambda) \quad \text{central rapidity}$$

How are we going to do it?

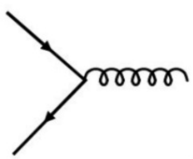
- Just compute collinear and soft interaction separately



SCET Feynman rule embedded in CGC

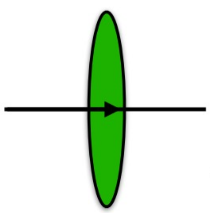
- Use CGC interaction in momentum space (Jamal, Farid, Raju, ...)

Collinear Interactions $\bar{n}\cdot A = 0$



$$= i g_s t^a \left(n^\alpha + \frac{\not{q}_\perp^\alpha \not{\gamma}_\perp}{\bar{n}\cdot q} + \frac{\not{\gamma}_\perp \not{q}_\perp'^{\alpha}}{\bar{n}\cdot q'} \right) \frac{\not{\bar{n}}}{2}$$

Bad components in terms of the good components

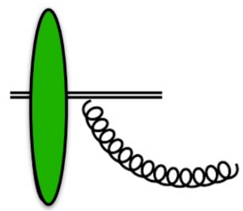


$$= \frac{\not{\bar{n}}}{2} \int db_\perp\, W_{ij}(b_\perp)\; 2(2\pi)\delta(p^+ - p'^+)$$

$+\dots$

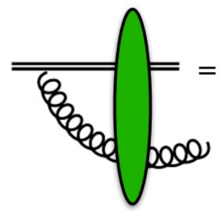
Soft Current

rapidity regulator Chiu, et al. 2011



$$= -g_s \mathbf{T}^a \frac{n^\alpha}{n\cdot k} \boxed{\left(\frac{\nu}{k_\perp}\right)^{\frac{\eta}{2}} e^{-\frac{\eta}{2}|y|}}$$

Nothing but eikonal approximation



$$= i \frac{g_s \mathbf{T}^b}{(4\pi)^{1-\epsilon}} \mu^{2\epsilon} \nu^{\frac{\eta}{2}} e^{-\frac{\eta}{2}|y|} \frac{\Gamma[1-\epsilon-\eta/4]}{\Gamma[1+\eta/4]} \int dx_\perp\, r_\perp'^{\alpha} \left[\frac{r_\perp'^{2}}{4}\right]^{-1+\epsilon+\eta/4} W_{ab}(x_\perp) e^{ik_\perp\cdot x_\perp}$$

Can be viewed as additional sources of soft radiations

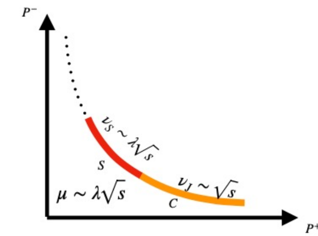
Kang, XL, 1910.10166

Courtesy of Liu

Results of collinear and soft contributions

- Use so-called rapidity regulator to regularize the rapidity divergence

Power counting



Collinear contribution

$$-\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)+\ldots$$

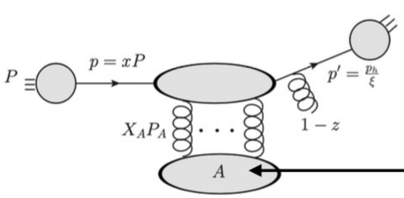
- Reproduce exactly the NLO using LFPT
- Regulator to turn rapidity divergence to pole (forward)
- Generate the rapidity scale for the collinear sector, $\nu_J \sim p^+$

Soft contribution

$$\delta(1-z)\left(\frac{2}{\eta}+\ln\frac{\nu^2}{p_\perp^2}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)$$

$$+\delta(1-z)\frac{\alpha_s}{\pi}\frac{N_C}{2}\frac{1}{\pi}\left(\frac{\ln(r_\perp'^2 p_\perp^2/c_0^2)}{r_\perp'^2}+\frac{\ln(r_\perp''^2 p_\perp^2/c_0^2)}{r_\perp''^2}+\frac{2r'_\perp\cdot r''_\perp}{r_\perp'^2 r_\perp''^2}\ln\frac{r''_\perp r'_\perp p_\perp^2}{c_0^2}\right)_+\mathscr{F}$$

- Poles (forward + backward)
- Reproduce the kinematic constraints, automatically arises
- Rapidity scale for the soft sector, $\nu_S \sim p_\perp$



$$\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p_\perp^2/p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)$$

- Remaining rap. pole to be absorbed (cancelled) by the small-x distribution
- Rap. scale arises naturally, $\nu \sim p_\perp^2/p^+ \sim x_A P_A \sim e^{-Y_A}$
- Will produce the BK equation

See also, Liu, Ma, Chao 1909.02370

Another amazing fact

- The finite contribution from the soft modes exactly reproduce the contributions as found via “kinematic constraint”
  - Now it becomes systematic, not just put by hand, and can be easily generalized to higher orders
- Thus this new factorization formalism in the small-x regime, with both collinear and soft modes, would be really nice

Threshold logarithm: negative contribution

- Log structure at NLO in the factorized formalism
  - First term – DGLAP (can be threshold PDF/FF), third term – BK evolution
  - Second term is interesting: large Nc, a single logarithm, so-called threshold log
    - When z is large (forward is the case), then it will be important

More logs to worry about

$$\frac{\mathrm{d}^2\hat{\sigma}^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 \cancel{P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)}$$

resummed

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\boxed{\frac{1}{z}\tilde{P}_{i\to i}(z)}e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\right.$$

$$\left.+\cancel{\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+}\right\}W_{aa'}(x_\perp)+\dots$$

resummed

$$\tilde{P} \sim \frac{2}{(1-z)_+} \qquad \frac{p_{h,\perp}}{\sqrt{s}}e^y < z < 1$$

When $1-z$ is effectively small, the threshold contributions could be large and breaks down the fixed order results.

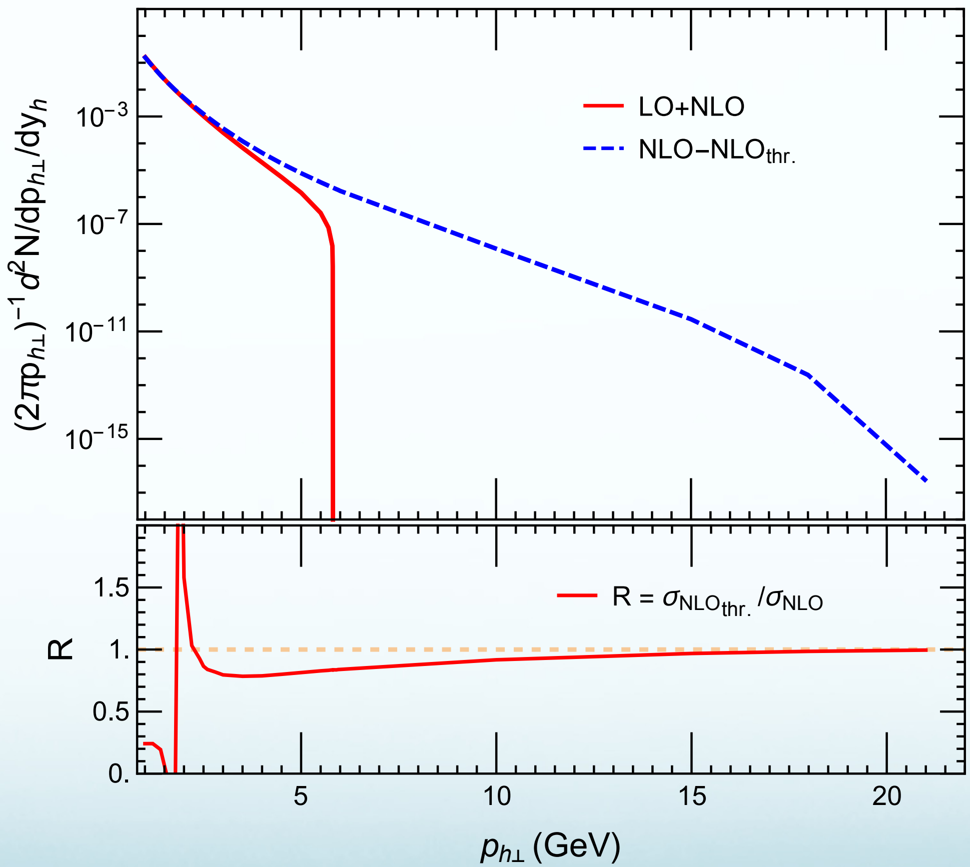
(Xiao, Yuan, 19, Kang, Liu, 19)

Also known as over-subtraction

(Iancu et al., 16, Ma, HardProbe2020)

Threshold log is the key

- Threshold log is the main issue for negative cross section
  - Remove it, the cross section will be positive in the entire pT region, up to the kinematic limit



Kang, Liu, Liu, 2004.11990

Threshold resummation

- Standard threshold resummation is well established in the pQCD community
  - Threshold PDFs/FFs: Mellin transform (Vogelsang, Sterman), Laplace transform (SCET)
  - Threshold TMDs: Neill 16, Wouter 16, Kang, Ding, Zhao, 2007.14425
- Threshold in the small-x regime is not easy
  - It is related to the rapidity logs
  - At the momentum, we only know how to do it at leading logarithmic (LL) level

QCD resummation

- Within our factorization, it is due to the rapidity evolution

More logs to worry about

$$\frac{\mathrm{d}^2\hat\sigma^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto \cancel{-\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)}$$

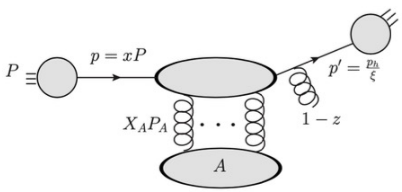
resummed

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\boxed{\frac{1}{z}\tilde P_{i\to i}(z)}e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\right.$$

$$\left.\cancel{+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+}\right\}W_{aa'}(x_\perp)+\dots$$

resummed

$$\tilde P \sim \frac{2}{(1-z)_+} \qquad \frac{p_{h,\perp}}{\sqrt{s}}e^y < z < 1$$



No real energetic collinear radiations allowed. Collinear momentum occurs only in the virtual loops

When $1-z$ is effectively small, the threshold contributions could be large and breaks down the fixed order results.

(Xiao, Yuan, 19, Kang, Liu, 19)

Also known as over-subtraction

(Iancu et al., 16, Ma, HardProbe2020)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_h\mathrm{d}^2p_{h\perp}} = \sum_{i,j=g,q}\frac{1}{4\pi^2}\int\frac{\mathrm{d}\xi}{\xi^2}\frac{\mathrm{d}x}{x}zxf_{i/P}(x,\mu)D_{h/j}(\xi,\mu)$$

$$\times\int\mathrm{d}^2b_\perp\mathrm{d}^2b'_\perp\, e^{ip'_\perp\cdot r_\perp}$$

$$\times\left\langle\langle\mathcal{M}_0(b'_\perp)|\,\boldsymbol{\mathcal{J}}(z,\mu,\nu,b_\perp,b'_\perp)\boldsymbol{\mathcal{S}}(\mu,\nu,b_\perp,b'_\perp)|\mathcal{M}_0(b_\perp)\rangle\right\rangle_\nu .$$

$$\boldsymbol{\mathcal{J}}(z)\to\boldsymbol{\mathcal{J}}_{thr.} \qquad \boldsymbol{\mathcal{S}}\to\boldsymbol{\mathcal{S}}_{thr.}(z)$$

Contains only loops

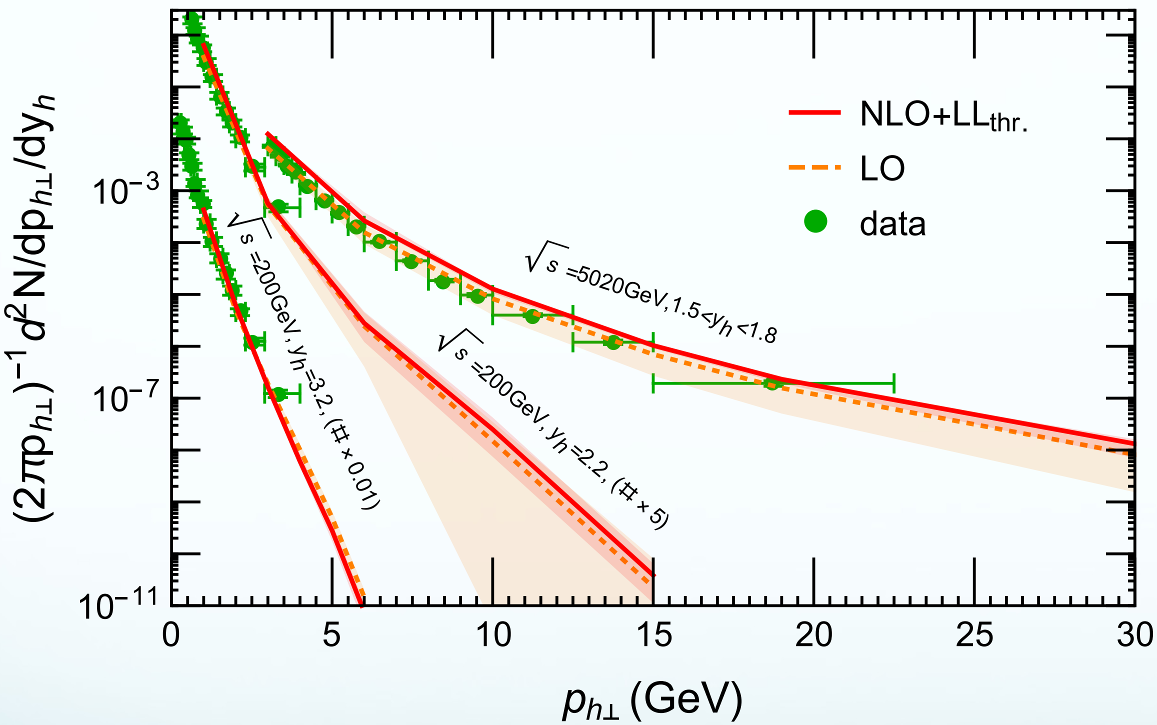
$$U_{J_{thr.}}U_{S_{thr.}} = \exp\left[-\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}x_\perp}{\pi}\left(\ln\frac{\nu_S}{\nu_J}I_{BK,r}\right.\right.$$

$$\left.\left.+\ln\frac{X_f}{X_A}I_{BK}\right)\mathbf{T}_i^a\mathbf{T}_j^{a'}W_{aa'}(x_\perp)\right]$$

- Novel threshold resummation structure.
- CGC rap. scale choice can not resum threshold logs
- Dynamical scale X_f can be determined numerically to minimize the evolution.

Phenomenology: example

- Now in all pT region, we are positive



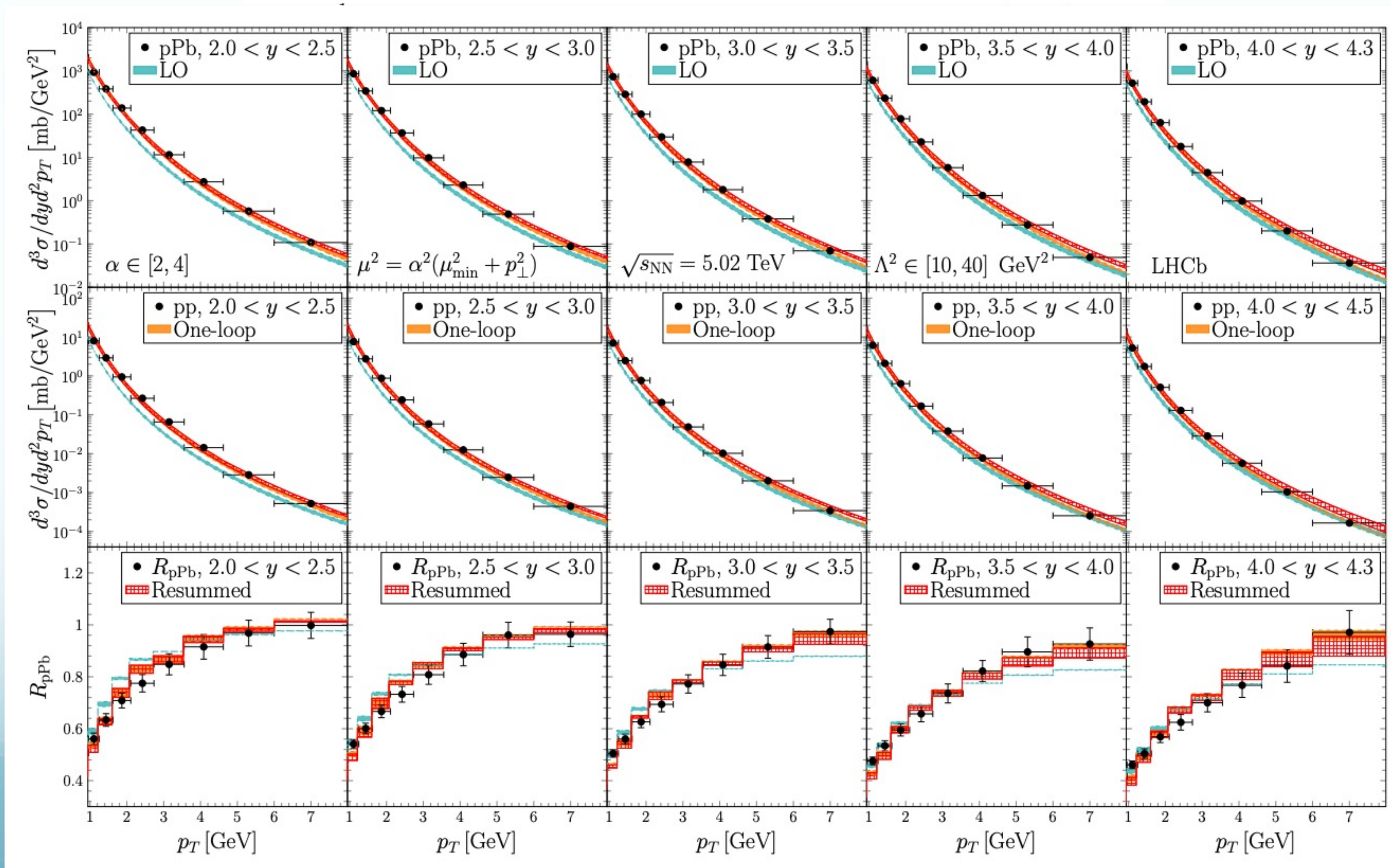
Kang, Liu, Liu, 2004.11990

Threshold resummation by Xiao et.al.

- Some subtle difference from us: Sudakov factor (color structure)

Pursuing the Precision Study for Color Glass Condensate in Forward Hadron Productions

Yu Shi,[1,2,*] Lei Wang,[2,†] Shu-Yi Wei,[1,3,‡] and Bo-Wen Xiao[4,§]



Summary

- By combining the advantages of SCET and CGC, we would be able to answer a lot of outstanding questions for small-x physics
  - We would be able to convert small-x physics to a high-precision physics
  - We would be able to make (hopefully) easier phenomenological studies for small-x observables
- Both are urgently needed in the era of EIC

Thank you!